

Quantum pump for counter-circulations in a spinor Bose-Einstein condensate

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We propose a pump scheme for quantum circulations, including counter-circulations for superposition states, of a spinor Bose-Einstein condensate. Our scheme is efficient and can be implemented within current experimental technologies and setups. It remains valid for non-classical atomic states, such as pseudo-spin squeezed states and maximal entangled N-GHZ or NooN states. Moreover, it is capable of transforming several enhanced sensing protocols relying on reduced fluctuations from quantum correlation and entanglement of atomic internal states to enhanced measurement of spatial interference and rotation.

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The existence of superposition states is a hallmark of quantum mechanics. Distinctly nonclassical properties such as quantum entanglement can arise for superpositions of more than one degree of freedom. With proper quantification schemes, entanglement, essentially ubiquitous in multi-party superposition states, is viewed as an important resource for quantum information processing and quantum computation. Generation of superposition states with high entanglement resource is a challenging experimental goal confronted many technique difficulties, not the least being decoherence of a superposition from interacting with environment and reservoirs. Atomic quantum gases, with its demonstrated long coherence time and controllability under many circumstances, are ideal candidates for investigating quantum coherence properties [1].

This Letter proposes a mechanism for generating superposition states of counter-circulations in a spinor atomic Bose-Einstein condensate (BEC), analogous to entangled states between atomic spin and its center of mass orbital rotation [2]. Recently, making use of the orbital angular momentum (OAM) of light, a controllable scheme was proposed [3, 4], capable of creating an arbitrary superposition of vortices and anti-vortices.

Generating vortices in a condensate using OAM from a Laguerre-Gaussian (LG) light beam was suggested long ago [5–9], and confirmed experimentally in various Raman coupling schemes [10–13]. An alternative, some times termed a vortex pump idea, is capable of continuously changing vorticity through repeated manipulations of external magnetic (B-) field. Since it was first proposed by Isoshima *et al.* [14], a number of studies [15–17] have focused on this simple protocol of creating a vortex through adiabatically flipping the bias B-field of an Ioffe-Pritchard trap (IPT). It has been faithfully demonstrated by several groups [18–21]. In recently proposed generalizations [22, 23], continuous vortex pumping is proposed using two sets of B-fields with different geometries to break the otherwise time reversal symmetry between the flip and back-flip (unflip) steps of each pump cycle. A radial x -bias in the back-flip step improves the operation efficiency, leading to a monotonically increasing vorticity with higher and higher rotations follow-

ing repeated flip and back-flip cycles [23]. While experimental challenges remain, especially for large vorticities from repeated pump cycles, every step of the continuous pump cycle has been demonstrated despite various compromises in adiabaticity, heating, spin flips, and decoherence associated with repeated controls of the B-field gradient. What remains unclear, however, is whether the B-field control protocols are capable of generating superposition states of counter-circulations, reminiscent of the so-called super counter flows [24].

In this Letter, we provide an affirmative answer that the B-field manipulating vortex pump protocols [22, 23] remain applicable to quantum superposition states, thus can be used to generate counter-circulation states. Furthermore, as we shall illustrate below through numerical simulations, the generalized quantum pump protocols remain effective for a host of quantum correlated states: such as single atom superpositions of counter-circulating states, many atom superpositions of counter-circulating states, spin squeezed, and maximally entangled or NooN-like counter-circulating states.

Before presenting the details, we describe briefly how an adiabatic flip of the axial z -bias B-field can be used to create counter circulating states. For an atom of hyperfine spin F in a 2D multipole B-field, there exists a conserved quantity $D_z = L_z - (Q - 1) \times F_z$ [23, 25] with $Q = 2$ ($Q = 3$) for a 2D quadrupole (hexapole) field. \vec{L} is the OAM of the atomic center of mass motion. An atom with no initial vorticity ($L_z = 0$) and in the $F_z = -\hbar$ state gains a circulating vorticity of $L_z = 2(Q - 1)\hbar$ following an adiabatic flip of the axial bias to the internal spin state $F_z = \hbar$. The conservation of D_z also implies an atom with no initial vorticity ($L_z = 0$) and in the $F_z = \hbar$ state is adiabatically flipped into the $F_z = -\hbar$ while gaining a circulating vorticity of $L_z = -2(Q - 1)\hbar$. Both scenarios can occur simultaneously, giving rise to a superposition of counter-circulating states, if trapping is facilitated by an externally applied optical trap instead of by the B-field itself [23].

For spin-1 ^{87}Rb atoms, the two internal states $|\uparrow\rangle \equiv |F = 1, M_F = 1\rangle$ and $|\downarrow\rangle \equiv |F = 1, M_F = -1\rangle$ can be coupled together with two-photon Raman process, thus any single atom superposition state of $\alpha\psi_{\uparrow}(\vec{r})|\uparrow\rangle +$

$\beta\psi_{\downarrow}(\vec{r})|\downarrow\rangle$ can be prepared with ease and precision, albeit in a trap. The flipping of the axial bias then creates a condensate state whereby every atom is in the superposition $\alpha\psi_{\uparrow}(\vec{r})e^{-2i(Q-1)\phi}|\downarrow\rangle + \beta\psi_{\downarrow}(\vec{r})e^{2i(Q-1)\phi}|\uparrow\rangle$. The respective mode function $\psi_{\uparrow,\downarrow}(\vec{r})$ is obtained from solving coupled Gross-Pitaevskii equations and ϕ is the azimuthal angle. Adopting the vortex pump protocol described previously [22, 23], or as described step by step in the paragraphs to follow, a superposition state with opposite high vorticity in each component is prepared after q pumping cycles: $\alpha\psi_{\uparrow}(\vec{r})e^{-i2q(Q-1)\phi}|\uparrow\rangle + \beta\psi_{\downarrow}(\vec{r})e^{i2q(Q-1)\phi}|\downarrow\rangle$. This same substitution from pump operation for a single atom $|\uparrow\rangle \rightarrow |\uparrow\rangle e^{-i2q(Q-1)\phi}$ and $|\downarrow\rangle \rightarrow |\downarrow\rangle e^{i2q(Q-1)\phi}$ is transmittable to many atom states: i.e., it can be used to generate circulation squeezing from spin squeezed states and maximal circulation entangled state with counter circulations $\alpha\psi_{\uparrow\ldots\uparrow}(\vec{r}_1, \ldots, \vec{r}_N)e^{-i2q(Q-1)\sum_i\phi_i}|\uparrow\ldots\uparrow\rangle + \beta\psi_{\downarrow\ldots\downarrow}(\vec{r}_1, \ldots, \vec{r}_N)e^{i2q(Q-1)\sum_i\phi_i}|\downarrow\ldots\downarrow\rangle$ from a NooN-like state, where ϕ_i represents the azimuthal angle of the i -th atom. A projective measurement of the internal spin along the orthogonal direction (to $|\downarrow\rangle$ and $|\uparrow\rangle$) interferences the two spatial amplitudes, giving rise to fringes along the azimuthal direction, which is sensitive to rotation as in the famous Sagnac effect [26].

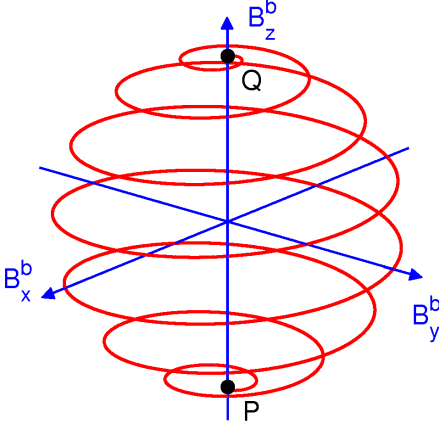


FIG. 1: (Color Online). The temporal trajectory of the total bias B-field. The back-flip begins with the bias along the $-z$ -axis, depicted by the point P. In the end, it is reset to the original direction, the point Q along the $+z$ -axis, before the first step, ready for the next pump cycle.

To facilitate experimental effort, we now discuss an improved back-flip step for the quantum pump protocol, utilizing both bias field controls along the transverse x - and y -axis as in a TOP [27]. The 2D quadrupole field (2DQT) of the IPT will be kept untouched during the operation. In addition to the many nice features as we illustrate below, this revised back-flip step is ideally suited for manipulating quantum superposition states. In the first step of each pump cycle, we still flip the bias of the IPT to create a $2F$ vortex or increase its vorticity by $2F$ [14–17]. For the second step, a previous protocol requires a radial x -axis bias [23, 25]. If the 2DQT were kept fixed,

the zero B-field center will move along opposite direction when the x -axis bias is increased. The axial symmetry is broken, as a result, atoms will follow to chase the low field region. The atomic spins thus cannot be effectively rotated along the vertical direction of local B-field, which was the reason to turn off the 2DQT before back-flipping the atomic spin [23, 25]. Based on extensively numerical simulations and analytical investigations of the adiabatic limits in TOP, we now find that the 2DQT can be kept in place, provided a rotating radial bias is introduced as in a TOP [27] when the axial z -bias is back-flipped. When both x - and y -biases are timed with the axial bias flip, we can reset the atomic spin and the IPT without adversely affecting the vorticity already accumulated.

As a typical implementation, in the first step we adiabatically flip the axial z -bias B_z^b of the IPT while maintaining the 2DQT. The time dependence of B_z^b was discussed previously [23]. The initial value for B_z^b is chosen to be much larger than the radial B-field so that all atoms are properly polarized. For a spin-1 condensate, initially all atoms are in the low-field seeking state, the averaged spin angular momentum per atom is $F_z \simeq -\hbar$. Flipping of the bias changes F_z to \hbar referenced to the initial positive z -axis. The flipped internal state remains weak field seeking because $F_z = -\hbar$ if the quantization axis were chosen to be along the flipped B-field direction in the $-z$ -axis. This first flip is the same as before [22, 23], which creates a vortex in a spin-1 condensate with an OAM $L_z = 2\hbar$, or increase its vorticity by $2\hbar$. In the improved second step as outlined above, the axial bias is back-flipped together with a rotating radial bias as illustrated in Fig. 1, which are affected if the three biases B_x^b , B_y^b , and B_z^b are tuned simultaneously according to

$$\begin{aligned} B_x^b(t) &= B_z^b(0) \sin\{\pi[1 - (t - T_1)/T_2]\} \cos(\omega_{\text{rf}}t), \\ B_y^b(t) &= B_z^b(0) \sin\{\pi[1 - (t - T_1)/T_2]\} \sin(\omega_{\text{rf}}t), \\ B_z^b(t) &= B_z^b(0) \cos\{\pi[1 - (t - T_1)/T_2]\}, \end{aligned} \quad (1)$$

where T_1 and T_2 are the durations for the first and second steps respectively, and ω_{rf} is the frequency for the transverse bias rotation. The transverse bias rotation is engineered to be along z -axis so that the axial symmetry is approximately maintained, and the OAM per atom L_z remains conserved. A typical B-field trap frequency is $\sim 10^2\text{Hz}$, while the Rabi frequency between the two internal states can be easily much larger $\sim 10^6\text{Hz}$, of the order of the Larmor frequency. A reasonable ω_{rf} is $\sim 10^3\text{Hz}$ or higher in order to assure adiabaticity as in TOP. At the end of the back flip step, all atoms are reset to as before the first step except for the vorticity gained. Cyclically repeating the two steps gives rise to the new vortex pump protocol, now with the 2DQT fixed.

Our quantum pump protocol is now demonstrated numerically for a spin-1 ^{87}Rb condensate with $N = 10^5$ ^{87}Rb atoms ($F = 1$) in an IPT with an optical plug $V_p(\rho, z, \phi) = U \exp(-\rho^2/2\rho_0^2)$. The actual trapping of atoms comes from the optical trap $V_o = M\omega_{\perp}^2(x^2 + y^2 + \lambda^2 z^2)/2$. Gravity is assumed along the minus z -

axis direction. For simplicity, atoms are confined to the ground state $\phi(z)$ in the z -direction. The effective coupled 2D Gross-Pitaevskii equations for $\psi(\vec{\rho}, t)$ defined by $\Psi(\vec{r}, t) = \psi(\vec{\rho}, t)\phi(z)$ becomes [28],

$$\begin{aligned} i\hbar \frac{\partial \psi_{\pm 1}}{\partial t} &= \left[H_0 + H_{\pm 1 \pm 1}^{\text{ZM}} + c_2^{(2\text{D})}(n_{\pm 1} + n_0 - n_{\mp 1}) \right] \psi_{\pm 1} \\ &\quad + c_2^{(2\text{D})} \psi_{\mp 1}^* \psi_0^2 + H_{\pm 1 0}^{\text{ZM}} \psi_0 + H_{\pm 1 \mp 1}^{\text{ZM}} \psi_{\mp 1}, \\ i\hbar \frac{\partial \psi_0}{\partial t} &= \left[H_0 + H_{00}^{\text{ZM}} + c_2^{(2\text{D})}(n_1 + n_{-1}) \right] \psi_0 \\ &\quad + 2c_2^{(2\text{D})} \psi_0^* \psi_1 \psi_{-1} + H_{01}^{\text{ZM}} \psi_1 + H_{0-1}^{\text{ZM}} \psi_{-1}. \end{aligned} \quad (2)$$

For the optical traps, we take $\lambda = 50$ and $\omega_{\perp} = 2\pi \times 60$ Hz, $U/\hbar = 2 \times 10^5$ Hz, and $\rho_0 = 5 \mu\text{m}$. The total B-field is $\vec{B} = B'(x\hat{x} - y\hat{y}) + B_x^b\hat{x} + B_y^b\hat{y} + B_z^b\hat{z}$, where the first term is the familiar 2DQT, which is fixed during the whole process, and is responsible for the geometrical phase structure of the vortex state. B' is chosen to be 5 G/cm. In the first step of each cycle, biases B_x^b and B_y^b are turned off, and the z -axis bias B_z^b is flipped according to the time dependence of Eq. (4) of [23] with $B_z^b(0) = 0.1$ G. In the second step, all three biases are programmed according to the time dependence of Eq. (1), with $T_1 = 45$ ms, $T_2 = 15$ ms, and $\omega_{\text{rf}} = 2\pi \times 5$ kHz.

The initial superposition state is prepared by imaginary time propagation of coupled Gross-Pitaevskii equations (2) starting from $\psi_G(|\uparrow\rangle_B + |\downarrow\rangle_B)$, with ψ_G a Gaussian wave function and the subscript B denotes local B-field quantization, under a fixed IPT $B'(x\hat{x} - y\hat{y}) + B_z^b(0)$. The two states $|\uparrow\rangle_B$ and $|\downarrow\rangle_B$ are essentially the same as $|\uparrow\rangle$ and $|\downarrow\rangle$ whenever the B-field bias $B_z^b(0)$ dominates. At the end of the imaginary time propagation, the state takes approximately the form $(\psi_{\uparrow}|\uparrow\rangle + \psi_{\downarrow}|\downarrow\rangle)/\sqrt{2}$, from which we then apply our quantum pump protocol in real time. In the first step of each cycle, the z -axis bias B_z^b is flipped from 0.1 G to -0.1 G, in 45 ms. A vorticity of $2\hbar$ ($-2\hbar$) is pumped into the component $|\uparrow\rangle$ ($|\downarrow\rangle$). The second step keeps the vorticity, resetting the axial bias and the atomic spins.

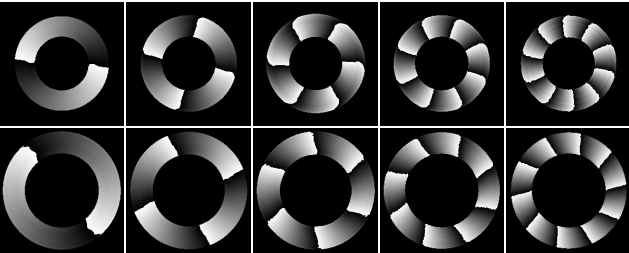


FIG. 2: The temporal development of the spatial phase structures from the quantum pump with a rotating transverse bias $\vec{B}_h = B'(x\hat{x} - y\hat{y}) + B_x^b\hat{x} + B_y^b\hat{y} + B_z^b\hat{z}$. The first(second) row shows the $|\uparrow\rangle$ ($|\downarrow\rangle$) component at the end of the first flips, or in the middle of each cycle at 45, 105, 165, 225, and 285 ms. The maximum for B_x^b , B_y^b , and B_z^b are 0.1 G, 0.1 G, and 0.1 G, respectively. White (black) color denotes phase $-\pi$ (π).

Figure 2 illustrates the intended time development of

the spatial phase structure. The first (second) row shows the $|\uparrow\rangle$ ($|\downarrow\rangle$) component at the end of the first flips, or in the middle of each pump cycle at various times respectively. Despite the presence of a 2DQT, the vortex pump works nicely as predicted. It increases and decreases pump respectively circulations for the initial $|\downarrow\rangle$ and $|\uparrow\rangle$ components, or it enlarges their respective vorticities during the first step of each pump cycle. The vorticities gained are maintained during the second steps of each cycle while the system resets. The whole pump scheme is now considerably simpler, and from the experimental point of view, can be implemented with mature technologies, particularly at laboratories with TOPs already in place.

The above discussion and numerical demonstration concerns the quantum pump protocol applied to a condensate of all atoms in the same two state superposition $[\psi_{\uparrow}(\vec{r})|\uparrow\rangle + \psi_{\downarrow}(\vec{r})|\downarrow\rangle]^{\otimes N}/2^{N/2}$. As was alluded to earlier, our quantum pump also applies to more general superposition states, e.g., spin squeezed states and maximally entangled N-GHZ or NooN state. Because of the limitations in simulating spatial wave functions for a many atom state, as an illustration, we consider the simple case of an entangled two atom state $[\psi_{\uparrow\uparrow}(\vec{r}_1, \vec{r}_2)|\uparrow\uparrow\rangle + \psi_{\downarrow\downarrow}(\vec{r}_1, \vec{r}_2)|\downarrow\downarrow\rangle]/\sqrt{2}$, which provides the crucial link to generalize our pump protocol to many atom systems. After the first step of bias flip, the initial state evolves into $[\psi_{\uparrow\uparrow}e^{-i2(\phi_1+\phi_2)}|\downarrow\downarrow\rangle + \psi_{\downarrow\downarrow}e^{i2(\phi_1+\phi_2)}|\uparrow\uparrow\rangle]/\sqrt{2}$ according to our quantum pump protocol. It further evolves into $[\psi_{\uparrow\uparrow}e^{-i2(\phi_1+\phi_2)}|\uparrow\uparrow\rangle + \psi_{\downarrow\downarrow}e^{i2(\phi_1+\phi_2)}|\downarrow\downarrow\rangle]/\sqrt{2}$ during the second step of back-flip, with the same spatial phase structure intact as at the end of the first pumping cycle. The numerical simulations use the same parameters as in Fig. 2, but with $\omega_{\perp} = 2\pi \times 100$ Hz, $\lambda = 30$, and $\rho_0 = 2 \mu\text{m}$. For simplicity, we choose to neglect interactions between the two spin-1 ^{87}Rb atoms.

Figure 3 demonstrates the actual temporal development for the phase structures of the two atom wave function $\psi_{\sigma_1\sigma_2}(\vec{r}_1, \vec{r}_2)$. To simplify the graph, we fix one atom at the radius corresponding to the maximum density $\sum_{\sigma_1\sigma_2} \int d\vec{r}_2 |\psi_{\sigma_1\sigma_2}(\vec{r}_1, \vec{r}_2)|^2$, and specific azimuthal angles ϕ_1 where the spin state for one atom is $M_F = \sigma_1$ while $M_F = \sigma_2$ for the other. The first (second) row reveals perfect vortical structure for the the $|\uparrow\uparrow\rangle$ ($|\downarrow\downarrow\rangle$) components after the first step of bias flip. Similar results are obtained for the $|\downarrow\downarrow\rangle$ ($|\uparrow\uparrow\rangle$) components at the end of each pumping cycle. We thus show for two atomic states $|\uparrow\uparrow\rangle$ ($|\downarrow\downarrow\rangle$), each one gains a vorticity $-2\hbar$ ($2\hbar$) irrespective how the other atom evolves. The vorticity gained is further shown to remain preserved after the back-flip of the bias. Since the two atoms we adopt are identical, the vorticity gained in the two atoms will be the same during the whole protocol, consistent with our previous discussion. Not surprisingly, these results can still be understood by considering the conservation of $D_z = L_z - F_z$.

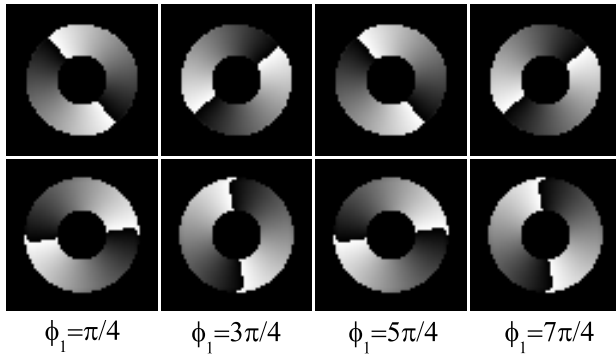


FIG. 3: The vortical phase structures for two atoms with one fixed at azimuthal angle $\phi_1 = \pi/4, 3\pi/4, 5\pi/4$, and $7\pi/4$ from the left to the right column. The first (second) row shows the spatial distribution for the remaining phase of the wave function component $|\uparrow\uparrow\rangle$ ($|\downarrow\downarrow\rangle$) after the first step of bias flip. White (black) color denotes phase $-\pi$ (π).

The two atomic internal states we illustrate here are very different; state $|F = 1, M_F = -1\rangle$ is low-field seeking, while $|F = 1, M_F = 1\rangle$ is high-field seeking. More generally, any two states with opposite M_F can be used as long as they can both be confined within the geomet-

rical pole region of a 2DQT. When the two states are of similar magnetic properties, like the two weak field seeking states $|F = 2, M_F = 1\rangle$ and $|F = 1, M_F = -1\rangle$ of ^{87}Rb atoms [29], additional optical traps become less relevant. Moreover, our protocol is checked to work equally well for superpositions with different amplitudes α and β , and for superfluid mixtures of different atomic species [30]. The validity for our pump protocol can be confirmed through interfering the two amplitudes [3, 12].

In conclusion, we propose a practical and effective quantum vortex pump scheme capable of increasing/decreasing vortical circulations of an arbitrary superposition for two state atoms. An immediate application concerns the generation of superposition states exhibiting nonclassical circulation properties such as from spin squeezed states with enhanced signal to noise ratios upon on suitable measurement. Our work maps enhanced sensing capabilities of correlated many atom states to rotational sensing, and shines new light on a number of interesting topics from sensitive interferometers to test of rotational equivalence principle [31].

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